Comment on "Instabilities of Isotropic Solutions of Active Polar Filaments"

In their recent Letter [1], Liverpool and Marchetti investigate the stability of homogeneously and isotropically distributed polar filaments in solution and interacting with molecular motors. They conclude that "At low filament density the system develops a *density* instability, while at high density entanglement drives the instability of *orientational* fluctuations." They have shown that the real part of the relevant branch of the eigenvalue spectrum has a positive curvature at the wave number k_b, where the two branches $\lambda_+(k)$ and $\lambda_-(k)$ merge in Fig. 2(b) of Ref. [1]. In this case, however, the growth rate is always negative at k_b, $-A = \lambda(k_b) < 0$ and a positive curvature $B = d^2\lambda(k)/dk^2$ at $k \gtrsim k_b$ opens according to the leading order expansion $\lambda = -A + B(k - k_b)^2$ the option of an instability as indicated by the dashed line in Fig. 1, but higher order contributions, such as $\propto k^4$, which are not included in Ref. [1], could keep $\lambda(k)$ in the negative range for all values of k, as shown by the solid line in Fig. 1. Therefore, only for a positive growth rate $\lambda(k_m)$ at its maximum does the isotropic state become unstable against a finite wave number perturbation.

For this purpose the expansion with respect to powers of ∇_r in Eq. (9) of Ref. [1] must not be terminated at the second order as in Ref. [1] but higher order terms have to be taken into account.

Including these terms, a dispersion relation follows as described by the solid line in Fig. 1, whose maximum $\lambda(k_m)$ becomes only positive by further increasing the parameter α beyond the curve $\alpha_o(\rho)$ in Fig. 2. The dotted curve $\alpha_s(\rho)$ is determined as in Ref. [1] by the change of sign of $d^2\lambda/dk^2|_{k=k_b}$ and lies considerably below the true instability curve $\alpha_o(\rho)$. This is not only a remarkable difference but we emphasize that the higher order term is not just a correction, but really essential for the prediction of the orientational instability in Ref. [1].

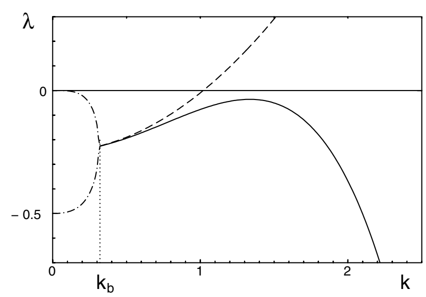

FIG. 1. The growth rate $\lambda(k)$ is shown as a function of the wave number k. The dashed and the dash-dotted lines correspond to the case in Ref. [1], neglecting higher order terms in k. Higher order terms stabilize the homogeneous state, because the maximum $\lambda_+(k_m) < 0$ of the solid line at k_m is negative. The parameters are $\rho = 4$, $\alpha = 10.3$, and $\beta = 5$.

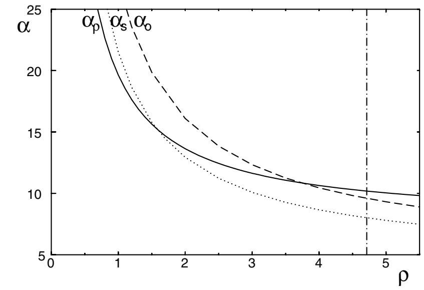

FIG. 2. $\alpha_\rho(\rho)$ describes the threshold for the density fluctuations, $\alpha_s(\rho)$ where the curvature of $\lambda(k)$ becomes positive at k_b and $\alpha_o(\rho)$ where the isotropic state becomes unstable against orientational fluctuations. The vertical line marks the threshold density for the nematic isotropic transition [2,3], where the theory of Ref. [1] breaks down due to the omission of the nematic order parameter. Accordingly the range of a pure orientational instability may become rather small.

Also the important questions of whether the instability with a critical wave number parallel or perpendicular to the filament orientation is preferred as well as the dependence of the wavelength of the emerging patterns on parameters can again only be answered by the determination of k_m and $\lambda(k_m) = 0$, and therefore by including higher order terms.

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