## Comment on "Entrainment of a Spatially Extended Nonlinear Structure under Selective Forcing"

In their recent letter [1], M. Henriot et al. investigate entrainment of spatially forced electroconvection in nematic liquid crystals, both experimentally by using a novel optical forcing technique and theoretically in terms of a model. However, the model that they use in Ref. [1] does not fit the conditions of the experiment described in the same work.

Immediately above threshold of a supercritical bifurcation to a spatially periodic and stationary pattern with a wavenumber $\vec{k}_0 = (k_x, 0)$, the pattern may be described as a product of a fast varying periodic function $\exp(ik_x x)$ and a slowly varying envelope $A(x,y,t)$ as $\vec{u}(x,y,z,t) = A(x,y,t)\exp(ik_x x)\vec{u}_0(z,t) + cc.$. Small deviations of the field $\vec{u}(x,y,z,t)$ from the periodicity $\exp(ik_x x)$, i.e. long–wavelength modulations, are described by the envelope $A(x,y,t)$. An example is electroconvection, where the components of the vector $\vec{u}_0(z,t)$ describe the various fields of the respective system.

Along with this common spirit, Henriot et al. use for the envelope $A(x,y,t)$ in Ref. [1] the model equation

$$\partial_t A = [\varepsilon + \delta\varepsilon \exp(i(\vec{q}_f \cdot \vec{r} - \phi))]\, A + (\partial_x^2 + P\partial_y^2)A - \partial_y^4 A - |A|^2 A\,. \qquad (1)$$

Without the multiplicative forcing term $\delta\varepsilon \exp(i(\vec{q}_f \cdot \vec{r} - \phi))A(x,y,t)$ this equation is similar to the universal amplitude equation that is valid close to the so–called Lifshitz-point, cf. Refs. [4,5] and which occurs for instance in electroconvection in nematic liquid crystals [2,3]. However, compared to Refs. [4,5] the contribution $iZ\partial_x\partial_y^2 A$ with the real number $Z$ has been neglected in Eq. (1).

The wavevector $\vec{q}_f = \vec{k}_f - \vec{k}_0$ in Eq. (1) and as introduced in Ref. [1] is the difference between the wavenumber of the external forcing, $\vec{k}_f$, and the wavenumber of the convection rolls, $\vec{k}_0$. In the experiments described in Ref. [1] the modulation wavevector and that of the convection rolls have a similar modulus, $|\vec{k}_f| \simeq |\vec{k}_0|, |\vec{k}_{OR}|$ and therefore, the model in Eq. (1) does not apply to their experimental situation, as explained in the following.

For systems that are spatially forced by a modulation wavenumber $\vec{k}_f \simeq n\vec{k}_0$, there is according to Coullet's work [6] for each value of $n = 1, \ldots, 4$ a generic equation, which takes in one spatial dimension the form

$$\partial_t A = \varepsilon A + \partial_x^2 A - |A|^2 A + \delta\varepsilon e^{in\Delta q x} A^{*(n-1)}\,, \qquad (2)$$

with the small wavenumber detuning $n\Delta q = |\,\vec{k}_f - n\vec{k}_0\,|$, $\vec{k}_f \parallel \vec{k}_0$ and $A^*$ is the conjugate complex of $A$. Since $A$ describes long wavelength modulations, $n\Delta q$ is small compared to $|\vec{k}_f|$ and $|\vec{k}_0|$. For $\vec{k}_f \simeq n\vec{k}_0$ the forcing term in the amplitude equation near the Lifshitz–point has the same form as in Eq. (2) and in Refs. [7]. Therefore, in the case $n = 1$, which corresponds to the experimental conditions in Ref. [1], the forcing term according to Eq. (2) does not include the envelope function $A(x,y,t)$, instead, the generic equation includes only an additive forcing term $\delta\varepsilon \exp(i(\Delta q x + k_y y)$ with $\Delta q,\ k_y \ll |\vec{k}_0|$ which is different to the multiplicative forcing term in Eq. (1).

In the nearly resonant case, $\vec{k}_f \simeq 2\vec{k}_0$, the forcing term in the Lifshitz equation is also multiplicative as $\delta\varepsilon \exp(i2\Delta q x)A^*(x,y,t)$, cf. Ref. [7], and not of the form $\delta\varepsilon \exp(i(\vec{q}_f \cdot \vec{r} - \phi))A(x,y,t)$ as in Ref. [1] and in Eq. (1). Therefore Eq. (1) neither applies to the experimental situation in Ref. [1] nor to the generic forcing case $\vec{k}_f \simeq 2\vec{k}_0$.

Nevertheless, a few patterns obtained in Ref. [1] with the model in Eq. (1) share similarities with patterns obtained with the Lifshitz equation already before in Refs. [7].

Walter Zimmermann, Martin Hammele and Stefan Schuler
Theoretische Physik, Universität des Saarlandes,
D-66042 Saarbrücken, Germany
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