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Comment

Comment on "On the threshold characteristics of the flexoelectric domains arising in a homogeneous electric field: The case of anisotropic elasticity" by Y.G. Marinov and H.P. Hinov

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Abstract. It is demonstrated that the key findings of the paper by Y.G. Marinov and H.P. Hinov, Eur. Phys. J. E **31**, 179-189 (2010), are in direct conflict with the general physical background of flexoelectric domains. This is caused by a methodological error in the theoretical analysis of the paper.

Flexoelectric domains (flexodomains) which appear in experiments as stripe patterns have been first described by Bobylev and Pikin [1]. They are observed in a planarly aligned nematic layer (parallel to the x-y plane) of thickness d, when an electric potential, U, larger than a threshold U_c, is applied along the z-direction. Flexodomains are characterized by a specific spatial variation of the locally preferred axis of the uniaxial nematics, *i.e.*, of the director $\mathbf{n}$: the planar basic director configuration $\mathbf{n}_0 = \hat{\mathbf{x}}$ is modified for $|z| < d/2$ by a z-dependent distortion $\delta\mathbf{n} = (0, \delta n_y, \delta n_z)$ in the form of splay ($\delta n_z \neq 0$) and twist ($\delta n_y \neq 0$). In addition $\delta\mathbf{n}$ is spatially periodic along the y-axis with the critical wave number q_c.

The existence of flexodomains requires a balance of the dielectric, elastic and flexo torques on the director [2]. The first one tends to stabilize (destabilize) the planar basic state when the dielectric anisotropy $\epsilon_a = \epsilon_\parallel - \epsilon_\perp$ is negative (positive). The two dielectric permittivities $\epsilon_\parallel$, $\epsilon_\perp$ characterize the dielectric tensor in nematics. The impact of the stabilizing elastic torques is measured by two positive elastic constants k_{11} (splay) and k_{22} (twist). The parametrization $k_{11,22} = k_{av}(1 \pm \delta k)$ with $k_{av} = (k_{11} + k_{22})/2$ and $-1 < \delta k < 1$ is convenient. Nonzero flexo torques, destabilizing in the present case, necessitate a splay distortion ($\delta n_z \neq 0$) accompanied with a certain amount of twist ($\delta n_y \neq 0$). Their strength is determined by the parameter combination $\delta e = |e_1 - e_3|$ of the flexoelectric coefficients e_1, e_3.

The paper of Marinov and Hinov [3] (MH) addresses for the first time the theory of flexodomains for arbitrary δk (in [1] only the special case $\delta k = 0$ was considered). As one central result of their analysis (see eq. (34) in MH), the authors claim the non-existence of flexodomains for the elastic constants outside the interval $1/3 < k_{22}/k_{11} < 3$ (corresponding to $|\delta k| < 1/2$ in our notation). This restriction looks rather peculiar and has motivated us to examine the calculations in MH in more detail. The authors have started from the correct well-known equations for δn_z and δn_y (eqs. (6) in MH). Their solution, however, suffers from a methodological error. The basic equations (eqs. (6) in MH) are of the form of coupled linear ordinary differential equations in z with constant coefficients. Instead of solving them by using the standard textbook ansatz $\delta n_y(z), \delta n_z(z) \propto \exp(\lambda z)$, the authors of MH have applied a sequence of matrix manipulations. This method, first proposed in [4,5], has been also used by the authors of MH in [6,7]. In all these publications the fact that the procedure does not work for z-dependent matrices has been overlooked: the transformation of eq. (12) into eq. (17) with the use of eq. (14) in MH makes use of the transformation $\hat{\mathbf{V}}^{-1}\mathbf{I}\frac{\mathrm{d}^2}{\mathrm{d}z^2}\hat{\mathbf{V}} \to \mathbf{I}\frac{\mathrm{d}^2}{\mathrm{d}z^2}$, only valid for a constant matrix $\hat{\mathbf{V}}$. In the present case, however, this matrix depends in general on z via the parameter $\alpha \propto \delta k\, z$ defined in eq. (11) in MH. Thus the eqs. (6) are not correctly solved except for the special case $\delta k = 0$ already considered in [1]. As a consequence the general analysis of flexodomains in MH and also in [6,7] does not hold.

In fact, the incorrectness of the findings in MH can be already demonstrated on the grounds of general physical considerations. In this way one obtains the phase diagram of flexodomains in the $(\epsilon_a, \delta k)$ plane shown in fig. 1. The generic features of the existence regime of flexodomains become most transparent when concentrating at first on the well-known case $\delta e = 0$, where only the

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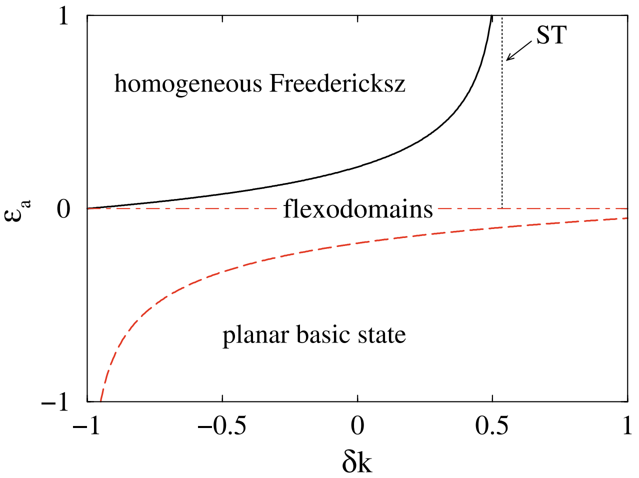

Fig. 1. Schematic illustration of the upper and lower limit curves, $\epsilon_a^u(\delta k)$ (solid) and $\epsilon_a^l(\delta k)$ (dashed), respectively of the existence range of flexodomains in the $(\epsilon_a, \delta k)$-plane (arbitrary units for ϵ_a). $\epsilon_a^u(\delta k)$ diverges at $\delta k \approx 0.535$ (dotted line ST), while $\epsilon_a^l(\delta k) \to -\infty$ for $\delta k \to -1$ (for details see text). In the absence of flexo torques ($\delta e = 0$) the planar state exists everywhere below the dot-dashed line $\epsilon_a = 0$. For $\epsilon_a > 0$ the homogeneous Freedericksz state is replaced by the splay-twist Freedericksz state to the right of the line ST (dotted).

dielectric and the elastic torques compete. For $\epsilon_a < 0$ (below the dot-dashed abscissa in fig. 1) the system remains for arbitrary U in the planar basic state, which is stabilized by both torques. For any $\epsilon_a > 0$, however, the destabilizing dielectric torque will eventually overcome the elastic torques with increasing voltage U: in the interval $-1 < \delta k < \delta k_{ST}$ with $\delta k_{ST} \approx 0.535$ ($k_{22}/k_{11} \approx 0.303$), *i.e.*, to the left of the dotted vertical line ST in fig. 1, the standard splay Freedericksz transition takes place [2]. The resulting distortion δn_z of the planar state depends only on z while $\delta n_y \equiv 0$. In contrast, as described by Lonberg and Meyer [8], in the interval $\delta k_{ST} < \delta k < 1$ (on the right of the line ST and above the dot-dashed line $\epsilon_a = 0$), the destabilization of the basic state happens in form of the splay-twist (ST) Freedericksz transition. In perfect analogy to flexodomains the resulting director distortions show nonzero δn_y, δn_z contributions, which are periodic along the y-direction.

Switching to finite δe, the destabilizing flexo torque enhances generically the tendency towards the splay-twist director variations. Thus the existence regime of the periodic splay-twist distortions for $\delta e = 0$, $\epsilon_a > 0$ and $\delta k > \delta k_{ST}$ in the upper right corner of fig. 1 must expand; it will then intervene between the homogeneous Freedericksz state and the planar state for arbitrary $-1 < \delta k < \delta k_{ST}$. Note that the special case $\delta k = 0$ [1] is consistent with this general scenario.

Let us discuss fig. 1 in more detail. For $\delta k > \delta k_{ST}$ and $\epsilon_a > 0$ the flexo torques yield only a minor modification of the periodic splay-twist director configuration already present for $\delta e = 0$. When $1 < \delta k < \delta k_{ST}$, however, the homogeneous Freedericksz configuration with $q_c = 0$ transforms continuously into the flexodomains with finite q_c along the "upper" transition curve $\epsilon_a = \epsilon_a^u(\delta k) > 0$. In the limit $\delta k \to \delta k_{ST}$ from below the line ST is approached by $\epsilon_a^u(\delta k)$ in the limit $\epsilon_a \to \infty$. Furthermore, the function $\epsilon_a^u(\delta k)$ has to decrease strictly with decreasing δk. While a finite δn_z is already provided by the dielectric torque, the increase of k_{22} blocks more and more the necessary twist distortion (δn_y) of the director.

For $\epsilon_a < 0$ the flexo torques may also overcome the stabilizing dielectric and elastic torques allowing for flexodomains in this regime as well. As is well known, the strength of the flexo torque increases linearly with the wave number q of the flexodomains. Thus when decreasing ϵ_a their critical wave number q_c has to become larger in order to over-compensate the increasing stabilizing effect of the dielectric torque ($\propto |\epsilon_a|$). Eventually q_c and also U_c diverge at the "lower" transition line $\epsilon_a^l(\delta k)$ separating the flexodomains and the planar basic state. Like $\epsilon_a^u(\delta k)$ the function $\epsilon_a^l(\delta k)$ decays monotonically with δk until it diverges when $\delta k \to -1$ ($k_{11} \to 0$) (see fig. 1). The reason is that the finite splay distortion, a crucial ingredient of flexodomains, is less and less blocked by the corresponding elastic torque $\propto k_{11}$.

The qualitative phase diagram for flexodomains in fig. 1 is in strong disagreement with the results of MH. Besides the unphysical limitation $|\delta k| < 1/2$ (eq. (34) in MH) for the existence of flexodomains, their upper limit function $\epsilon_a^u(\delta k)$ (according to eq. (35) in MH) would not decay but is strictly increasing with decreasing δk until it diverges at $\delta k = -1/2$.

Our general, qualitative considerations in this comment have been confirmed by the standard treatment of the underlying equations (eqs. (6) in MH) in a recent publication [9]. Here one finds detailed discussions of U_c and q_c in dependence on the material parameters. In addition the application of an ac-voltage and the competition with patterns arising from the electrohydrodynamic instability have been discussed.

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